Stability of premixed gaseous flames propagating in Hele-Shaw cells

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Introduction

The stability of a premixed flame propagating in a Hele-Shaw cell is investigated. The first effort on the problem was due to Joulin and Sivashinsky [1], who expanded the classical hydrodynamic Darrieus-Landau model by including the wall effects through an Euler-Darcy law in the frame of the flame-sheet approximation. As a result, instabilities associated with the transport process in the flame were neglected. Kang et al. [2] extended the problem numerically by making use of a 2D compressible reactive Navier-Stokes formulation within the Poiseuille flow assumption. The present work performs the linear stability analysis of a steady planar flame propagating between two adiabatic parallel plates by including the Darrieus-Landau (due to density change across the flame front), Rayleigh-Taylor (due to buoyant forces) and diffusive-thermal (due to unequal rates of thermal to molecular diffusion) effects. The problem formulation is based on the asymptotic limit when the ratio of the plates separation to the thermal flame thickness, given by the parameter $a = h/\delta_T$, is sufficiently small.

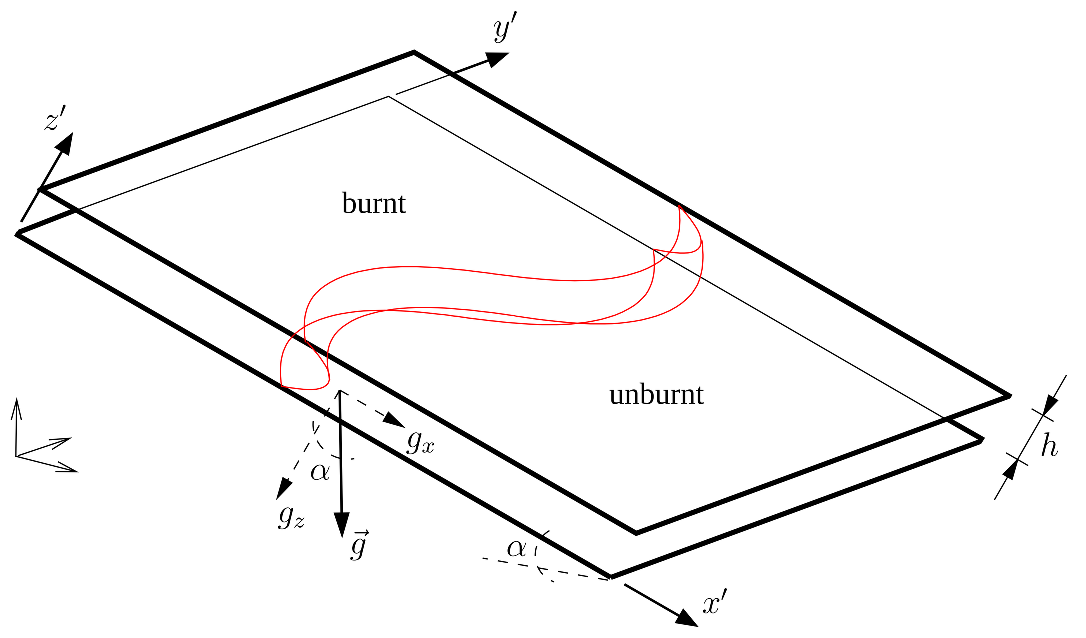

Fig. 1: Sketch of the Hele-Shaw cell with cell gap of size h and inclined at an angle α with the horizontal.

Quasi-2D formulation

We assume a quasi-isobaric flame propagation in the positive x' direction. The system is open to the atmosphere at the burnt side end and closed to the unburnt side end so that the flame propagates into a nominally quiescent gas. The system can be inclined at an arbitrary angle α with the horizontal. The chemical reaction is modeled by a global irreversible step of the form $Fuel \to Products$, strictly valid for fuel-lean mixtures. The adiabatic flame speed S_L and the thermal flame thickness $\delta_T = \mathcal{D}_T/S_L$, with $\mathcal{D}_T$ the thermal diffusivity of the mixture, are used as the reference scales for appropiate non-dimensionalization, together with the properties of the fresh unburnt mixture, in the form

$$
\begin{aligned}
(x, y, z) &= (x'/\delta_T, y'/\delta_T, z'/h), \quad t = t' S_L/\delta_T,\\
(u, v, w) &= (u'/S_L, v'/S_L, w'/(a\, S_L)), \quad \rho = \rho'/\rho_u,\\
p &= a^2(p' - p_{\rm atm})/(12\, Pr\, \rho_u S_L^2),\\
\theta &= (T' - T_u)/(T_{ad} - T_u), \quad Y = Y_F/Y_{F_u}.
\end{aligned}
$$

When the asymptotic limit $a = h/\delta_T \ll 1$ is employed, the flow properties are averaged across the cell gap and the governing equations, in a reference frame moving with the flame, $x \to x - u_f t$, are reduced to the quasi-2D form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho (U_x - u_f)}{\partial x} + \frac{\partial \rho U_y}{\partial y} = 0, \tag{1}$$

$$\rho \frac{\partial \theta}{\partial t} + \rho (U_x - u_f) \frac{\partial \theta}{\partial x} + \rho U_y \frac{\partial \theta}{\partial y} = \nabla^2 \theta + \omega, \tag{2}$$

$$\rho \frac{\partial Y}{\partial t} + \rho (U_x - u_f) \frac{\partial Y}{\partial x} + \rho U_y \frac{\partial Y}{\partial y} = \frac{1}{Le} \nabla^2 Y - \omega, \tag{3}$$

$$\rho(1 + q\theta) = 1, \tag{4}$$

where

$$\omega = \frac{\beta^2}{2 u_p^2 Le} (1+q)^2 \rho^2\, Y \exp\left\{ \frac{\beta(\theta - 1)}{1 + [q/(1+q)](\theta - 1)} \right\},$$

and

$$U_x = -\partial p/\partial x + G\rho \quad \text{and} \quad U_y = -\partial p/\partial y. \tag{5}$$

The parameters are: $\beta = E(T_{ad} - T_u)/R {T_{ad}}^2$, $q = \rho_u/\rho_b - 1 = (T_{ad} - T_u)/T_u$, $G = a^2 \delta_T\, g \sin\alpha/(12\, Pr\, S_L^2)$ and the Lewis number Le. The factor $u_p = S_L/U_L$, with $U_L = \sqrt{2 Le \mathcal{B} \rho_u \mathcal{D}_T/\beta^2}\, (\rho_b/\rho_u) \exp(-E/2RT_{ad})$, corresponds to the eigenvalue of the planar adiabatic problem, given elsewhere [3].

Stability formulation

The steady-state of temperature, mass fraction and flow velocity are perturbed in the form

$$f = f_0(x) + \epsilon f_1(x) \exp(iky + \lambda t), \tag{6}$$

where f stands for θ, Y, p, and U_x and U_y. Perturbations of the flame propagation velocity can be excluded without loss of generality. Here $\lambda \in \mathbb{C}$ (the real part of which represents the growth rate), k is the transverse wave number and ϵ is a small amplitude.

The linearized eigenvalue equations obtained when substituting Eq. (6) into an adequate combination of Eqs. (1)-(5) are reduced to

$$
\begin{aligned}
&\lambda \rho_0 \theta_1 - \frac{\partial \theta_1}{\partial x} + \rho_0 (U_{x1} + q\theta_1) \frac{\partial \theta_0}{\partial x} = \frac{\partial^2 \theta_1}{\partial x^2} - k^2 \theta_1\\
&\qquad\qquad + A\theta_1 + BY_1,\\
&\lambda \rho_0 Y_1 - \frac{\partial Y_1}{\partial x} + \rho_0 (U_{x1} + q\theta_1) \frac{\partial Y_0}{\partial x} = \frac{1}{Le} \left(\frac{\partial^2 Y_1}{\partial x^2} - k^2 Y_1 \right)\\
&\qquad\qquad - A\theta_1 - BY_1,\\
&\frac{\partial^2 U_{x1}}{\partial x^2} - k^2\, U_{x1} = k^2 G q \rho_0^2 \theta_1 +\\
&\qquad\qquad q\, \frac{\partial \left(\partial^2 \theta_1/\partial^2 x - k^2 \theta_1 + A\theta_1 + BY_1 \right)}{\partial x},
\end{aligned}
$$

where

$$
\begin{aligned}
A &= BY_0 \left\{ \frac{\beta}{[1 + [q/(1+q)](\theta_0 - 1)]^2} - \frac{2q}{(1 + q\theta_0)} \right\},\\
B &= \frac{\beta^2}{2 u_p^2 Le} (1+q)^2 \rho_0^2 \exp\left\{ \frac{\beta(\theta_0 - 1)}{1 + [q/(1+q)](\theta_0 - 1)} \right\}.
\end{aligned}
$$

Results

Figs. 2 and 3 show the dependence of the real part of λ with the wavenumber k for different values of the gravity factor G and for $Le = 1$ and $Le = 0.7$, respectively. In the figures we also include (in symbols) the calculated growth rate of a sinusoidally perturbed planar flame in the quasi-2D numerical simulations of Eqs.(2)-(5). The perturbation of the planar solution is incorporated in the simulations by superimposing a periodic disturbance of the temperature in the y direction in the form $\mathcal{A}_0 \sin(ky)$, where $\mathcal{A}_0$ is a small initial amplitude of the temperature (typically $\mathcal{A}_0 = 0.1$). This disturbance of the temperature is quickly translated to a sinusoidal amplitude of the flame front, which starts to grow exponentially with the time ($\mathcal{A} \sim \exp \lambda t$). If the growth of the amplitude maintains a sinusoidal shape with the time, the value of λ is positive and real and the natural logarithm of the ratio $\mathcal{A}/\mathcal{A}_0$ with the time gives the value of λ. For completeness, we also show in Fig. 2 (dashed lines) the growth rate of the hydrodynamic solution given by Joulin and Sivashinsky [1] for the limit $a \ll 1$. These curves correspond to the algebraic dispersion relation

$$\lambda = \frac{q(1 + q - G)}{2(1+q)}\, k, \tag{7}$$

indicating that flames with $G > 1 + q$ stabilize the planar solution. For $Le = 0.7$ gravity can stabilize the largest hydrodynamic wavelengths of wrinkling but an intermediate region of wavelengths (see $G = 10$ in Fig. 3).

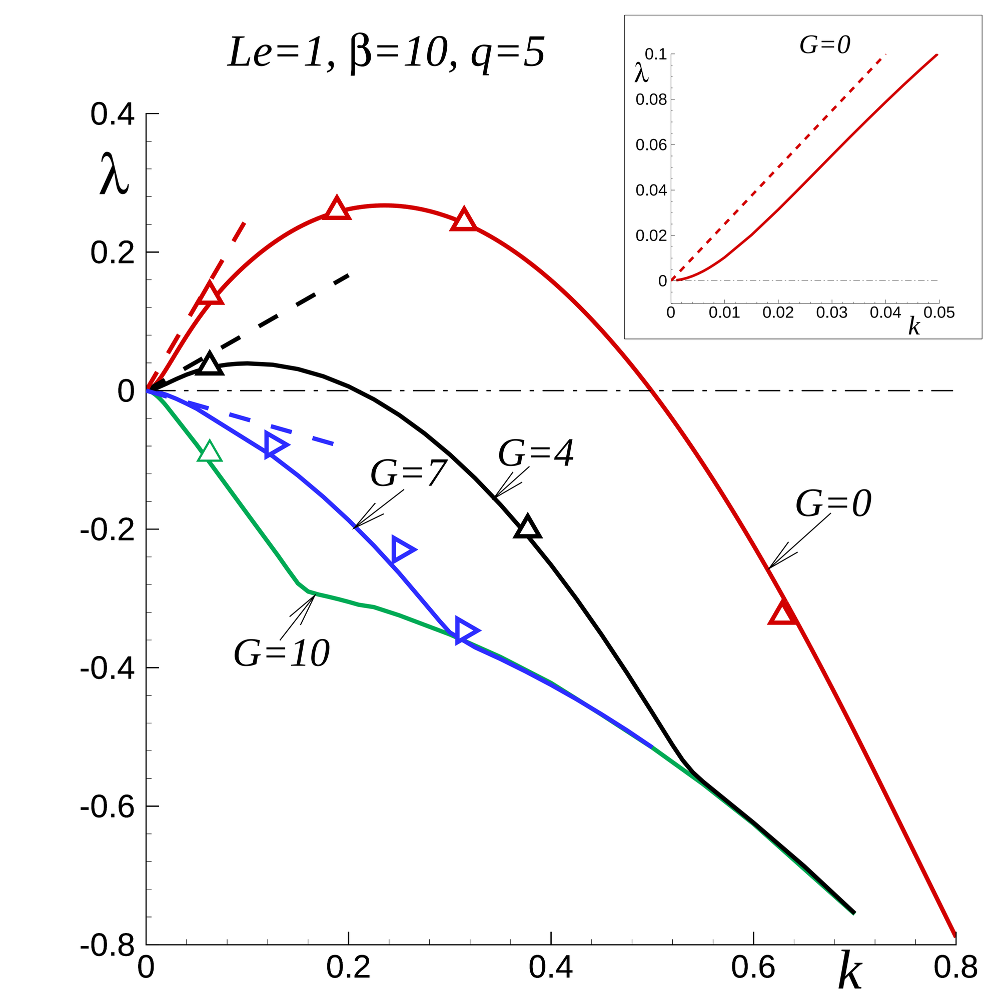

Fig. 2: The dependence of (real) λ on the wavenumber k calculated for $Le = 1$, $\beta = 10$, $q = 5$, and different values of G. Symbols correspond with the initial growth of a sinusoidally perturbed planar flame in quasi-2D numerical simulations. The dashed curve corresponds with the Joulin and Sivashinsky's hydrodynamic solution [1] in the limit $a \ll 1$.

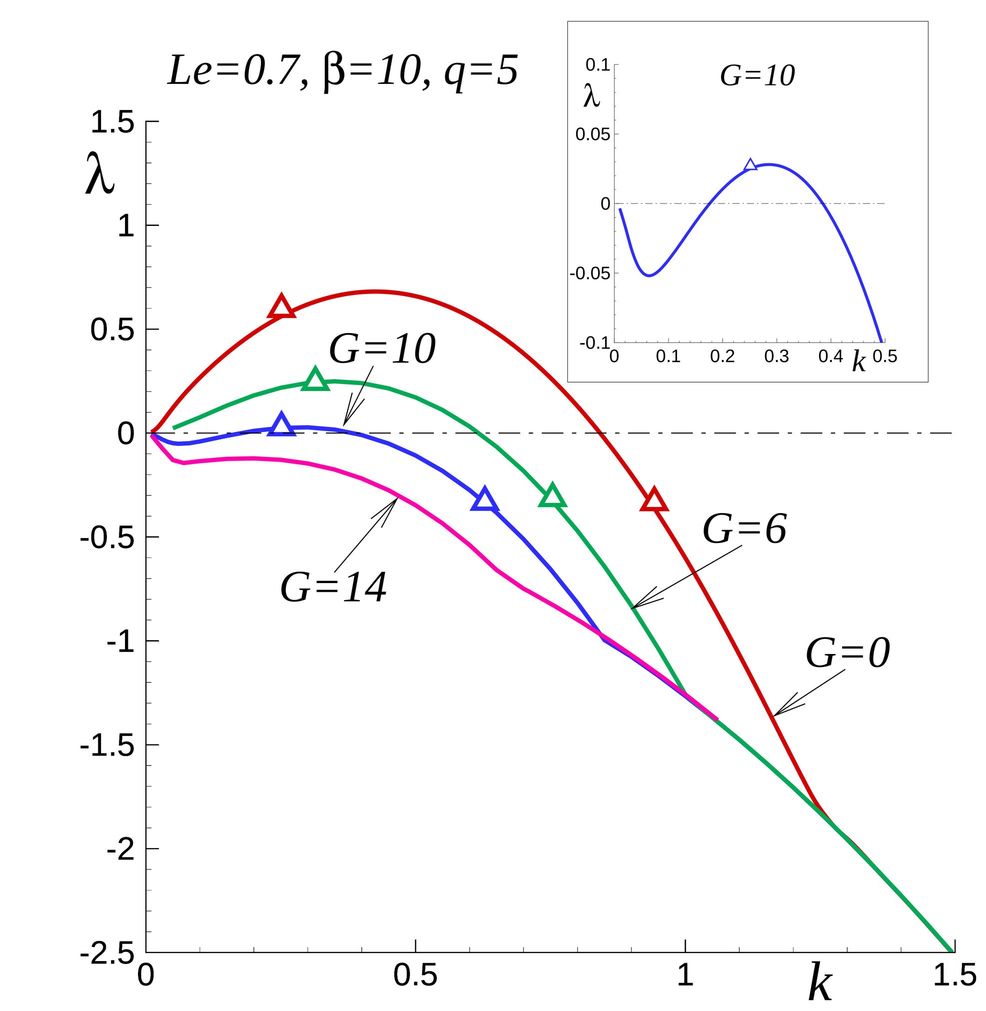

Fig. 3: The dependence of (real) λ on the wavenumber k calculated for $Le = 0.7$, $\beta = 10$, $q = 5$, and different values of G. Symbols correspond with the initial growth of a sinusoidally perturbed planar flame in quasi-2D numerical simulations

Future work

The effect of the Saffman-Taylor instability (due to viscosity change across the flame front) and small heat losses to the plates (of the order a) can be incorporated in the present formulation. The Saffman-Taylor mechanism is seen to introduce an important effect in confined geometries [4].

References

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