Legitimacy of the narrow-channel approximation for the study of flames propagating between two closely-spaced parallel plates

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Introduction

The propagation of laminar flame fronts between two adiabatic parallel plates separated a small distance h apart (often referred as a Hele-Shaw cell) is investigated using two numerical setups:

i. Recasting the conservation equations by taking the asymptotic limit $h/\delta_T \ll 1$, where δ_T is the thermal flame thickness, so that the mathematical problem reduces from a three-dimensional (3D) to a two-dimensional (2D) set of equations governed by Darcy's law [1, 2]. This transformation results in a drastic reduction of computational cost compared to the full 3D description.

ii. Performing 3D simulations directly.

The objetive is to determine the upper limit of the validity of narrow-channel approximation [1, 2] by comparing the long term evolution of (i) and (ii), that is, its flame topology and overall propagation rate.

Formulation

The governing equations are scaled using the unburnt state, the thermal flame thickness $\delta_T = \mathcal{D}_T/S_L$, and $\mathcal{D}_T/S_L^2$ as reference thermodynamic state, length and time scales, respectively. The dimensionless equations are formulated in a reference frame moving with the flame at velocity u_f, and read for:

i. narrow-channel aproximation

$$
\begin{aligned}
\partial\rho/\partial t + \nabla_{xy}\cdot[\rho(\mathbf{v}-u_f)] &= 0,\\
\mathbf{v} &= -Pr^{-1}\,\nabla_{xy}\,p,\\
\partial(\rho\theta)/\partial t + \nabla_{xy}\cdot[\rho\theta(\mathbf{v}-u_f)] &= \nabla_{xy}^2\theta + \omega,\\
\partial(\rho Y)/\partial t + \nabla_{xy}\cdot[\rho Y(\mathbf{v}-u_f)] &= Le^{-1}\,\nabla_{xy}^2 Y - \omega,\\
\rho(1+q\theta) &= 1,
\end{aligned}
$$

ii. 3D simulations

$$
\begin{aligned}
\partial\rho/\partial t + \nabla\cdot[\rho(\mathbf{v}-u_f)] &= 0,\\
\partial(\rho\mathbf{v})/\partial t + \nabla\cdot[\rho\mathbf{v}\cdot(\mathbf{v}-u_f)] &= -\nabla p + Pr\,\nabla(\nabla\cdot\mathbf{v}+\nabla\cdot\mathbf{v}^{\mathrm{T}}),\\
\partial(\rho\theta)/\partial t + \nabla\cdot[\rho\theta(\mathbf{v}-u_f)] &= \nabla^2\theta + \omega,\\
\partial(\rho Y)/\partial t + \nabla\cdot[\rho Y(\mathbf{v}-u_f)] &= Le^{-1}\,\nabla^2 Y - \omega,\\
\rho(1+q\theta) &= 1.
\end{aligned}
$$

The reaction rate is given by the Arrhenius expression

$$
\omega = \frac{\beta^2}{2Le\,u_p^2}(1+q)^2\rho^2\,Y\exp\left\{\frac{\beta(\theta-1)}{(1+q\theta)/(1+q)}\right\}.
$$

The computational domain is held fixed to $80\delta_T$ long, $40\delta_T$ wide and variable h thick (for the 3D cases) with kinetic and transport parameters fixed to $\beta = 10$, $q = 5$, $Le = 1$, and $Pr = 0.7$. Boundary conditions are periodic in the lateral y-domain, open end at $x \to -\infty$ and closed end at $x \to \infty$. Initial condition is a planar flame to which a small temperature perturbation $\Delta T(x,y) = \epsilon\exp(-|x-x_\omega|)\sin ky$ is added. The amplitude of the perturbation ϵ is of the order 10^{-2} times the adiabatic temperature, k is the wavenumber corresponding to the maximum linear growth rate [2], and x_ω is the position of the maximum reaction rate.

2D results - narrow channel approximation

The initially planar flame becomes curved due to hydrodynamic instabilities of the Darrieus-Landau (DL) type, as shown in Figs. 1 and 2. This results in an increased flame surface area and overall propagation rate.

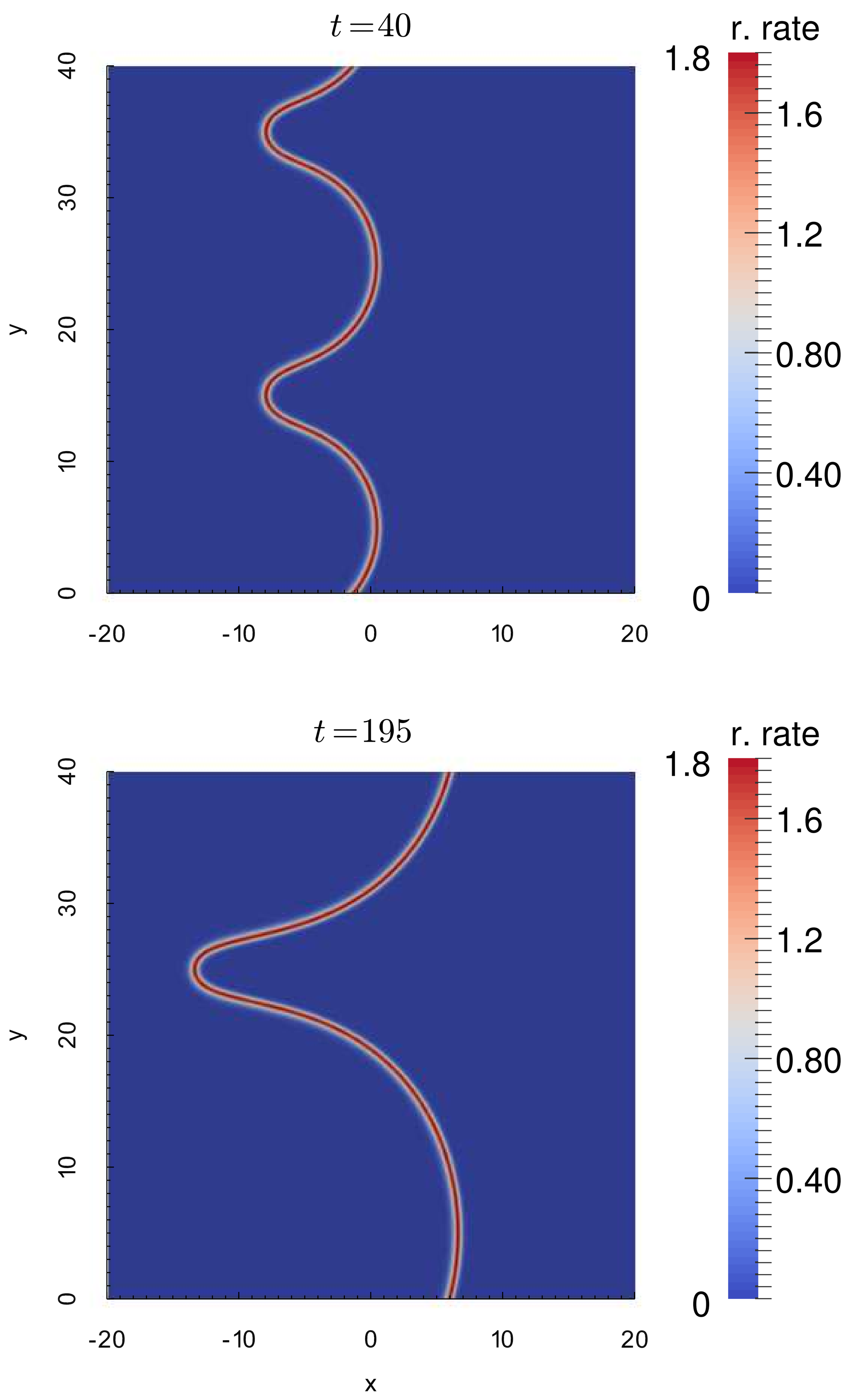

Fig. 1: Reaction rate field for $h/\delta_T \to 0$ showing the flame front curvature. Flame propagates from left to right.

3D results

3D simulations allows in-and out-of-plane gradients. For the maximum cell gap $h/\delta_T = 1$ investigated no flame curvature was observed in the third (z) dimension.

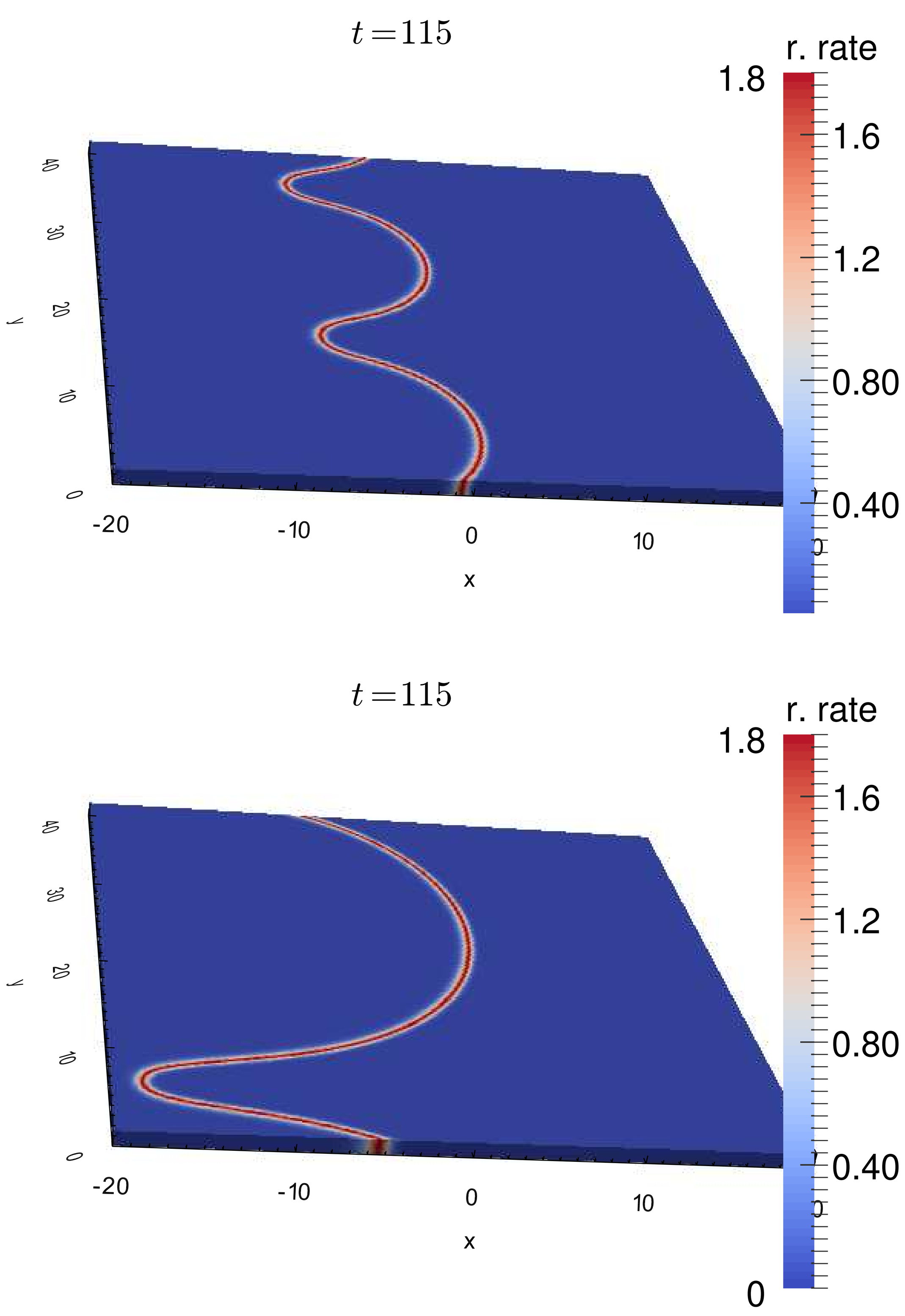

Fig. 2: Reaction rate field for $h/\delta_T = 1$ showing the flame front curvature. Flame propagates from left to right.

Propagation rates

The overall propagation rate is calculated using

$$
u_f = \frac{S_T}{S_L} = \frac{1}{L_y}\int_{-\infty}^{+\infty}\int_{y_{\min}}^{y_{\max}}\omega\,\mathrm{d}x\mathrm{d}y.
$$

Figure 3 shows the propagation rate as a function of time for $h/\delta_T = 0, 0.1$ and 1. At early times, the planar flame front destabilizes towards a two-cusps structure (corresponding with the wave number of the maximum linear growth rate). At late times, the flame wrinkles coalesce into a single cusp.

Preliminary results indicate that the narrow-channel approximation works well beyond its strict limit of validity, e.g. for $h/\delta_T \sim \mathcal{O}(1)$, and predicts well the flame dynamics and propagation rates.

Note that the effect of confinement ($h/\delta_T \sim 1$) results in overall propagation rates that are higher than those associated with unconfined flames, where $(S_T/S_L)_{\text{unconfined}} \approx 1.2$ [3].

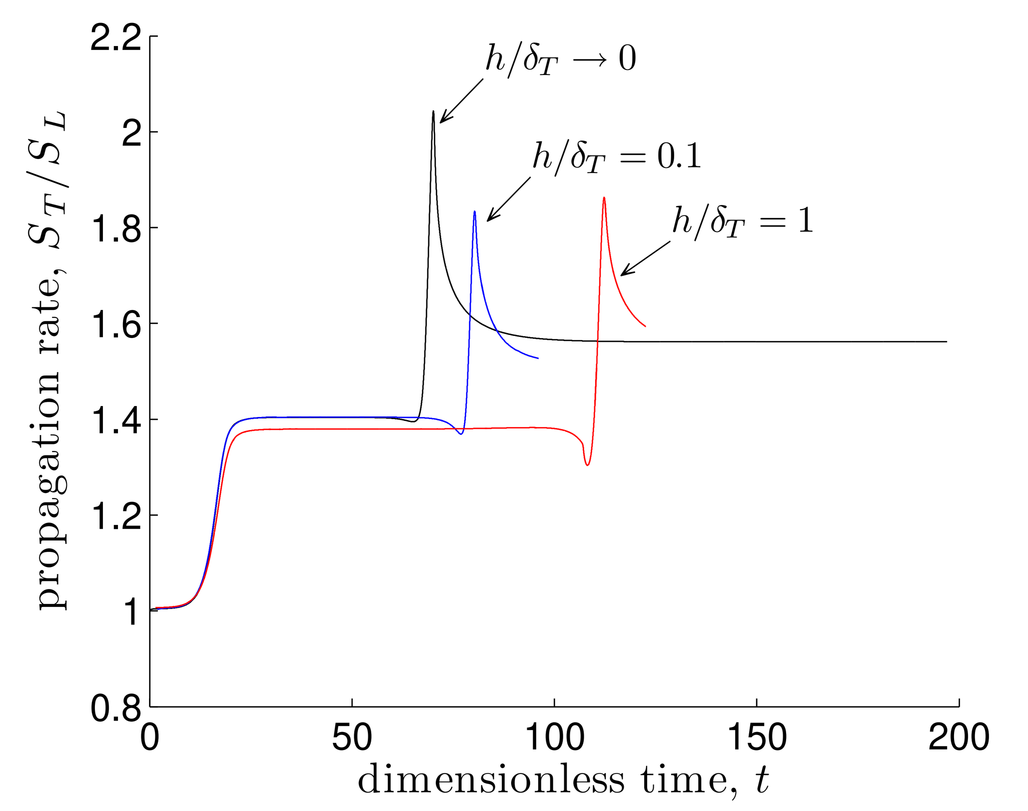

Fig. 3: Propagation rate as function of time for different h/δ_T.

Future work

To investigate the validity of the narrow-channel approximation when diffusive-thermal instabilities ($Le < 1$) are present, which may result in the emergence of non-symmetric flame shapes in the z-direction. We aim to demonstrate that premixed-gas flame dynamics in Hele-Shaw cells [4–6] can be studied using $h/\delta_T \to 0$ without loss of the essential dynamics.

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